Lesson 2: Recursive Formulas for Sequences

Classwork

**Example 1**

Consider Akelia’s sequence , , , , , ….

* 1. If you believed in patterns, what might you say is the next number in the sequence?
  2. Write a formula for Akelia’s sequence.
  3. Explain how each part of the formula relates to the sequence.
  4. Explain Johnny’s formula.

Exercises 1–2

1. Akelia, in a playful mood, asked Johnny: “What would happen if we change the ‘’ sign in your formula to a ‘’ sign? To a ‘’ sign? To a ‘’ sign?”
   1. What sequence does for and generate?
   2. What sequence does for and generate?
   3. What sequence does for and generate?
2. Ben made up a recursive formula and used it to generate a sequence. He used to stand for the th term of his recursive sequence.
   1. What does mean?
   2. What does mean?
   3. If and , write a possible recursive formula involving and that would generate and in the sequence.
   4. What does mean?
   5. What does mean?
   6. Would it necessarily be the same as ?
   7. What does mean?

**Example 2**

Consider a sequence given by the formula , where and .

* 1. List the first five terms of the sequence.
  2. Write an explicit formula.
  3. Find and of the sequence.

Exercises 3–6

1. One of the most famous sequences is the Fibonacci sequence:

….

, where ,, and .

How is each term of the sequence generated?

1. Each sequence below gives an explicit formula. Write the first five terms of each sequence. Then, write a recursive formula for the sequence.
   1. for
   2. for
2. For each sequence, write *either* an explicit or a recursive formula.
   1. , , , , , , …

* 1. , , , ,…

1. Lou opens a bank account. The deal he makes with his mother is that if he doubles the amount that was in the account at the beginning of each month by the end of the month, she will add an additional to the account at the end of the month.
   1. Let represent the amount in the account at the beginning of the th month. Assume that he does, in fact, double the amount every month. Write a recursive formula for the amount of money in his account at the beginning of the th month.
   2. What is the least amount he could start with in order to have by the beginning of the third month?

Problem Set

For Problems 1–4, list the first five terms of each sequence.

1. , where for
2. , where for
3. and for

Lesson Summary

**Recursive sequence**: An example of a *recursive sequence* is a sequence that (1) is defined by specifying the values of one or more initial terms and (2) has the property that the remaining terms satisfy a recursive formula that describes the value of a term based upon an expression in numbers, previous terms, or the index of the term.

An explicit formula specifies the th term of a sequence as an expression in .

A recursive formula specifies the th term of a sequence as an expression in the previous term (or previous couple of terms).

1. and for

For Problems 5–10, write a recursive formula for each sequence given or described below.

1. It follows a plus one pattern: , , , , , ….
2. It follows a times pattern: , , , , ….
3. It has an explicit formula of for .
4. It has an explicit formula of for .
5. Doug accepts a job where his starting salary is per year, and each year he receives a raise of .
6. A bacteria culture has an initial population of bacteria, and each hour the population triples in size.